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QGP and modified jet fragmentation

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Abstract. Recent progress in the study of jet modification in hot medium and its consequences in high-energy heavy-ion collisions is reviewed. In particular, I will discuss energy loss for propagating heavy quarks and the resulting modified fragmentation function. Medium modification of the parton fragmentation function due to quark recombination is formulated within finite temperature field theory and the implication for the search for a deconfined quark–gluon plasma is also discussed.

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1 Introduction

Jet quenching in high-energy nuclear collisions has been proposed as a good probe of the hot and dense medium [1,2] formed in ultra-relativistic heavy-ion collisions. The quenching of an energetic parton is caused by multiple scattering and induced parton energy loss during its propagation through the hot QCD medium. Recent theoretical estimates [3–7] all show that the effective parton energy loss is proportional to the gluon density of the medium. Therefore measurements of the parton energy loss will enable one to extract the initial gluon density of the produced hot medium. Because of color confinement in the vacuum, one can never separate hadrons fragmenting from the leading parton and particles materializing from the radiated gluons. The total energy in the conventionally defined jet cone in principle should not change due to induced radiation, assuming that most of the energy carried by radiative gluons remains inside the jet cone [8]. Additional rescattering of the emitted gluon with the medium could broaden the jet cone significantly, thus reducing the energy in a fixed cone. However, fluctuation of the underlying background in high-energy heavy-ion collisions makes it very difficult, if not impossible, to determine the energy of a jet on an event-by-event base with sufficient precision to discern a finite energy loss of the order of 10 GeV. Since high-p_T hadrons in hadronic and nuclear collisions come from fragmentation of high-p_T jets, energy loss naturally leads to suppression of high-p_T hadron spectra [2].

Since parton energy loss effectively slows down the leading parton in a jet, a direct manifestation of jet quenching is the modification of the jet fragmentation function, $D_{a\to h}(z,\mu^2)$, which can be measured directly in events in which one can identify the jet via a companion particle like a direct photon [9] or a triggered high-p_T hadron. This modification can be directly translated into the energy loss of the leading parton. Since inclusive hadron spectra are a convolution of the jet production cross section and the jet fragmentation function in pQCD, the suppression of inclusive high-p_T hadron spectra is a direct consequence of the medium modification of the jet fragmentation function caused by parton energy loss.

Strong suppression of high transverse momentum hadron spectra is indeed observed by experiments [10, 11] at the Relativistic Heavy-Ion Collider (RHIC) at the Brookhaven National Laboratory (BNL), indicating large parton energy loss in a medium with large initial gluon density. Shown in Fig. 4 are the nuclear modification factors $R_{AA}(p_T)$ for single hadron spectra as a function of the number of participant nucleons. The theoretical results are obtained from a LO pQCD parton model calculation [12] incorporating modified parton fragmentation functions due to parton energy loss,

$$\langle \Delta E\rangle \approx \pi C_a C_A \alpha_s^3 \int_{\tau_0}^{R_A} d\tau \rho(\tau)(\tau-\tau_0)\ln\frac{2E}{\tau\mu^2}. \quad (1)$$

The initial gluon density in the most central Au + Au collisions at $\sqrt{s}=200$ GeV was fixed by fitting the data. The centrality dependence shown is the consequence of the above parton energy loss, assuming the initial gluon density is proportional to the measured hadron multiplicity which in turn is approximately proportional to the number of participant nucleons. Such a calculation also has a definite prediction of the energy dependence of the hadron spectra suppression [13] that agrees well with the current collection of data at different energies [14].

Since the parton energy loss [see (1)] depends on the path length of the jet propagation, which in turn depends on the azimuthal angle with respect to the reaction

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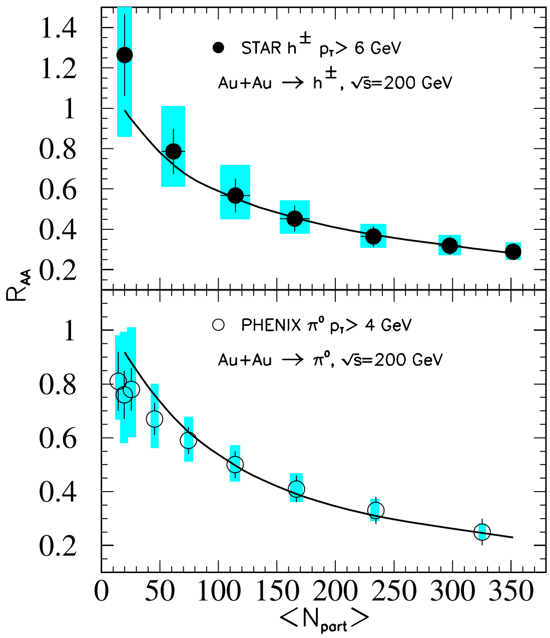

Fig. 1. The centrality dependence of the measured single inclusive hadron suppression [15,16] at high-p_{T} as compared to theoretical calculation with parton energy loss

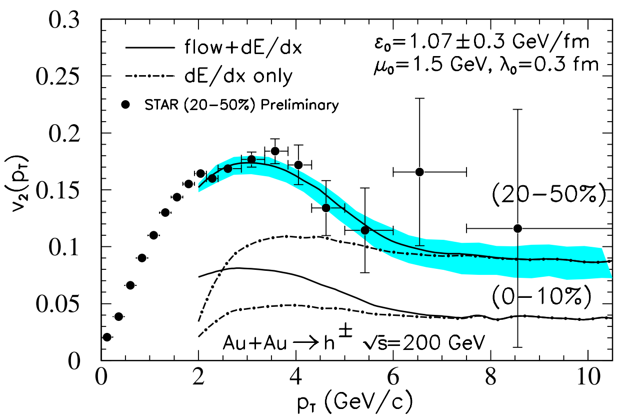

Fig. 2. Azimuthal anisotropy in Au+Au collisions as compared to the STAR [20] 4-particle cumulant result

plane in non-central collisions, the parton energy loss and modified fragmentation functions naturally lead to an azimuthal angle dependence of the hadron spectra suppression [17,18]. Indeed, the azimuthal angle distributions of high-p_{T} hadrons were found to display large anisotropy with respect to the reaction planes [19] of non-central Au + Au collisions. As shown in Fig. 5, the observed azimuthal anisotropy, characterized by the second coefficient of the Fourier transformation, v_2, can also be described well by the same pQCD parton model calculation.

The most striking measurement that is a direct manifest of jet quenching is the observed disappearance of the back-side high-p_{T} two-hadron correlation in azimuthal angle [21], which is characteristic of high-p_{T} back-to-back jets in $p+p$ collisions. Shown in Fig. 6 are back-side two high-p_{T} hadron correlations in Au + Au collisions with

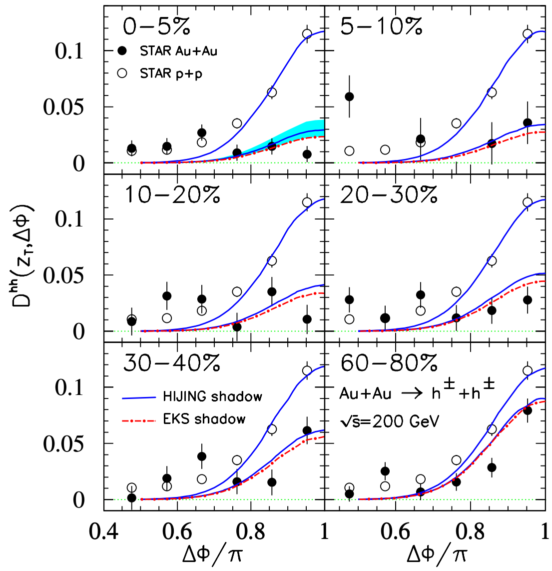

Fig. 3. Back-to-back correlations for charged hadrons with $p_{\mathrm{T}}^{\mathrm{trig}} > p_{\mathrm{T}} > 2\,\mathrm{GeV}/c$, $p_{\mathrm{T}}^{\mathrm{trig}} = 4$–$6\,\mathrm{GeV}/c$ and $|y| < 0.7$ in Au + Au (lower curves) and $p+p$ (upper curves) collisions as compared to the STAR[21] data

different centralities as compared to the same correlation in $p+p$ collisions. Though the back-side correlations in peripheral Au + Au collisions remain the same as in $p+p$ collisions, the peak gradually decreases and finally disappears in the most central Au + Au collisions. The curves are again from the same LO pQCD parton model calculation with parton energy loss. It describes both the magnitude of the suppression and the centrality dependence quite well.

Combining the above measurements of three different effects of parton energy loss and comparing with the measured jet quenching in deeply inelastic $e+A$ collisions, one can conclude that the initial gluon density reached in central Au + Au collisions at $\sqrt{s} = 200\,\mathrm{GeV}$ is about 30 times higher than in a cold Au nucleus [12,22], assuming the theoretical result that parton energy loss is proportional to the gluon density of the medium.

In this talk, I will review recent progress we have made in the study of modified jet fragmentation functions in medium and their applications to high-p_{T} hadron spectra and correlations in heavy-ion collisions. In particular, I will review heavy quark energy loss and modified fragmentation functions. Its unique features due to the heavy quark mass can help us to characterize the partonic nature of the observed jet quenching. I will also discuss di-hadron fragmentation functions since these address the hadron correlation within a jet and how it will also be modified by the parton energy loss. Finally, I will discuss the formulation of modified fragmentation functions due to quark recombination during the hadronization of the jet in a thermal medium. The implication on the search for deconfined quark–gluon plasma will also be discussed.

2 Modified heavy quark fragmentation function

In the study of parton energy loss, formation time for the radiated gluon plays an essential role. Because of the LPM interference, gluons with formation time longer than the length of a finite medium or the mean-free path in an infinitely large medium will be suppressed. Such formation time is only relative to the propagation of the leading parton. Therefore, gluon radiation from a heavy quark is normally shorter than that from a light quark because of the smaller velocity of the heavy quark. For radiated gluons with transverse momentum $\ell_\perp$ and z fractional momentum, the formation time is [23]

$$\tau_{\rm f} = \frac{2z(1-z)E}{\ell_{\rm T}^2 + (1-z)^2 M^2}, \qquad (2)$$

where M is the quark mass. Therefore, one should expect the LPM effect to be significantly reduced for intermediate energy heavy quarks. In addition, the heavy quark mass also suppresses gluon radiation amplitude at small angles [24] relative to that off a light quark. Both mass effects will lead to a reduced heavy quark energy loss compared to that of a light quark. The most significant consequence of the reduced formation time due to heavy quark mass is the change of the length dependence of the quark energy loss. The non-Abelian LPM effect due to suppression of gluon radiation with long formation time leads to a quadratic length dependence of the total energy loss. For a slow heavy quark, however, the dependence will become linear because of the short formation time and absence of the LPM interference. Shown in Fig. 4 are the numerical results of the nuclear size R_A dependence of charm quark fractional energy loss in DIS off a cold nucleus, rescaled by $\widetilde{C}(Q^2)C_A\alpha_{\rm s}^2(Q^2)/N_C$, for different values of $x_{\rm B}$ and Q^2. One can clearly see that the R_A dependence is quadratic for large values of Q^2 or small $x_{\rm B}$ (large initial quark energy) when the mass of the quark is negligible. The dependence becomes almost linear for small Q^2 or large $x_{\rm B}$. The charm quark mass is set at $M = 1.5\,{\rm GeV}$ in the numerical calculation.

One can similarly calculate the nuclear modification of the heavy quark fragmentation as in the case of a light quark [22]. Shown in Fig. 5 is the ratio of the modified charm quark fragmentation into D mesons to the vacuum fragmentation function. One can see that the modification due to the parton rescattering and induced gluon radiation in a nucleus for heavy quarks is quite different from light quarks [22, 25]. This is mainly caused by the form of the heavy quark fragmentation function in vacuum which peaks at large z. Because of the multiple scattering and induced gluon radiation, the position of the peak of the modified fragmentation function is effectively shifted to a smaller value of z. As a consequence, the heavy quark fragmentation function remains unchanged, or even slightly enhanced for a large range of fractional momentum z. The modification only becomes significant and the fragmentation function is suppressed at large z above the position of the peak. This is in sharp contrast to the case of modified light quark fragmentation functions which deviate from the vacuum form in a very large range of z.

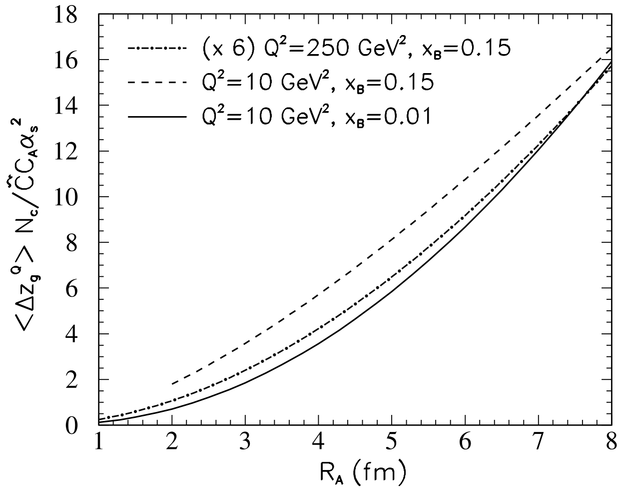

Fig. 4. The nuclear size, R_A, dependence of charm quark energy loss for different values of Q^2 and $x_{\rm B}$

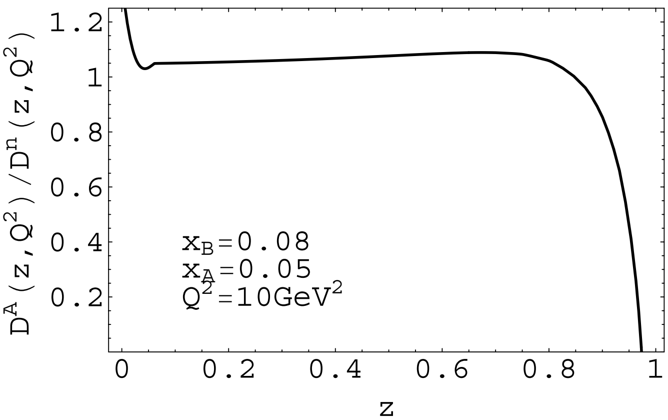

Fig. 5. Modification factor for the charm quark fragmentation function in a nucleus. The value $x_A = 0.05$ corresponds to a nucleus with a radius $R_A = 4.25\,{\rm fm}$

3 Non-Abelian feature of jet quenching

Another non-Abelian feature of the parton energy loss is its dependence on the color representation of the propagating parton. The energy loss for a gluon is 9/4 times larger than for a quark. One can investigate the consequences of this non-Abelian feature in the flavor dependence of the high-$p_{\rm T}$ hadron suppression [26]. In the mean time, we can also study the effect of the non-Abelian parton energy loss on the energy dependence of the inclusive hadron spectra suppression [27]. One can exploit the well-known feature of the initial parton distributions in nucleons (or nuclei) that quarks dominate at large fractional momentum (x) while gluons dominate at small x. Jet or large $p_{\rm T}$ hadron production as a result of hard scatterings of initial partons will be dominated by quarks at large $x_{\rm T} = 2p_{\rm T}/\sqrt{s}$ and by gluons at small $x_{\rm T}$. Since gluons lose

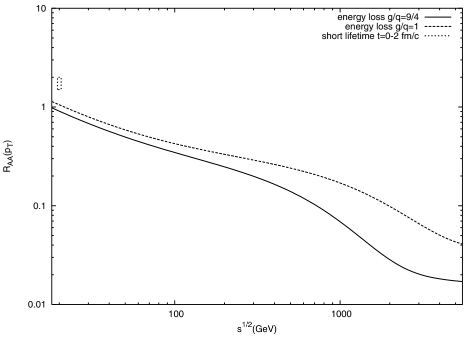

Fig. 6. Nuclear modification factor R_{AuAu} for neutral pions as a function of the collision energy at fixed $p_{\mathrm{T}} = 6\,\mathrm{GeV}$ in most central collisions (with centrality 10%). Here we compare the QCD energy loss and a non-QCD one where the energy loss is identical for quarks and gluons

9/4 more energy than quarks, the energy dependence of the large (and fixed) p_{T} hadron spectra suppression due to parton energy loss should reflect the transition from quark-dominated jet production at low energy to gluon-dominated jet production at high energy. Such a unique energy dependence of the high-p_{T} hadron suppression can be tested by combining $\sqrt{s} = 200\,\mathrm{AGeV}$ data with lower energy data or future data from LHC experiments.

To study the sensitivity of hadron spectra suppression to the non-Abelian parton energy loss, we calculate the single hadron spectra and the suppression factor with two different parton energy losses: one for the QCD case where the energy loss for a gluon is 9/4 times as large as that for a quark, i.e. $\Delta E_g/\Delta E_q = 9/4$; the other is for a non-QCD case where the energy loss is chosen to be the same for both gluons and quarks. Similarly, the average number of inelastic scatterings obeys $\langle \frac{\Delta L}{\lambda}\rangle_g / \langle \frac{\Delta L}{\lambda}\rangle_q = 9/4$ in the QCD case. For the non-QCD case we are considering, the above ratio is set to one. In order to demonstrate the difference between QCD and non-QCD energy loss, we compute the R_{AA} for neutral pions at fixed $p_{\mathrm{T}} = 6\,\mathrm{GeV}$ in central Au + Au collisions as a function of $\sqrt{s}$ from 20 AGeV to 5500 AGeV. Shown in Fig. 6 are the calculated results with both the QCD and non-QCD energy loss. In the calculation, we fix the initial gluon number density and quarks' mean-free path to fit the overall hadron suppression in the most central Au+Au collisions at $\sqrt{s} = 200\,\mathrm{GeV}$. For any other energy and centralities, we simply assume that the initial gluon number is proportional to the final measured total charged hadron multiplicity per unit rapidity. One can see that due to the dominant gluon bremsstrahlung or gluon energy loss at high energy the R_{AA} for the QCD case is more suppressed than the non-QCD case where the gluon energy loss is assumed to take an equal role as the quark.

Another interesting feature of the energy dependence of R_{AA} is the change of slope around $\sqrt{s} = 1300\,\mathrm{GeV}$. The rapid decrease of R_{AA} at $\sqrt{s} = 20$–$1300\,\mathrm{GeV}$ is mainly due to increased initial gluon density and also the change of the p_{T} slope of the jet production cross section with $\sqrt{s}$. As the energy loss increases, more jets produced inside the overlapped region are completely suppressed. Only those that are produced within an out-layer in the overlapped region will survive. This will be like surface emission with a finite depth. The suppression factor R_{AA} will then be determined by the width of the out-layer which is just the averaged mean-free path $\langle\lambda\rangle$. As a consequence, R_{AA} will then have much weaker $\sqrt{s}$ dependence. This effect was also seen by calculations in [28].

4 Modified dihadron fragmentation function

In addition to single inclusive hadron spectra from jet fragmentation, multiple hadron correlations have proven to be a useful measurement for characterizing the modification of the jet structure in a hot medium. For example, while the back-side two-hadron correlation is completely suppressed, the same-side correlation is observed to change little when the transverse momentum of the secondary hadron is large [21]. Such a same-side correlation is essentially given by dihadron fragmentation functions. It is important to investigate why such a dihadron fragmentation function changes little in the kinematic region of the experiments and whether it will be significantly modified when the secondary hadron is soft.

One can define the dihadron fragmentation functions in terms of the overlapping matrix between parton field operators and the final hadron states, similarly as the single hadron fragmentation functions. In the light-cone gauge, the dihadron fragmentation functions of a quark are defined as

$$
\begin{aligned}
&D_q^{h_1 h_2}(z_{h_1}, z_{h_2}) \\
&= \frac{z_h^4}{2 z_{h_1} z_{h_2}} \int \frac{\mathrm{d}^2 p_{h_1\perp}}{2(2\pi)^3} \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \delta\left(z_h - \frac{p_h^+}{p^+}\right) \int \mathrm{d}^4 x \mathrm{e}^{-\mathrm{i}p\cdot x} \\
&\times \mathrm{Tr}\left[\frac{\gamma^+}{2p_h^+} \sum_S \langle 0 |\psi(0)| S, p_{h_1}, p_{h_2}\rangle \right. \\
&\qquad \left. \times \langle p_{h_2}, p_{h_1}, S |\overline{\psi}(x)| 0\rangle\right] , \qquad (3)
\end{aligned}
$$

where $z_h = z_{h_1} + z_{h_2}$ and $p_h = p_{h_1} + p_{h_2}$. Like single hadron fragmentation functions, dihadron fragmentation functions also contain non-perturbative physics and thus are not calculable in pQCD. However, one can study their evolution with the energy scale within pQCD. The DGLAP evolution equations for dihadron fragmentation functions have been derived recently [29]. For the dihadron fragmentation function of a quark, it has the form

$$
\begin{aligned}
&\frac{\partial D_q^{h_1 h_2}(z_{h_1}, z_{h_2}, Q^2)}{\partial \log Q^2} \\
&= \frac{\alpha_{\mathrm{s}}}{2\pi}\left[\int_{z_{h_1}+z_{h_2}}^{1} \frac{\mathrm{d}z}{z^2} \gamma_{qq}(z) D_q^{h_1 h_2}\left(\frac{z_{h_1}}{z}, \frac{z_{h_2}}{z}, Q^2\right)\right. \\
&+ \int_{z_{h_1}}^{1-z_{h_2}} \frac{\mathrm{d}z}{z(1-z)} \\
&\quad \times \hat{\gamma}_{qq}(z) D_q^{h_1}\left(\frac{z_{h_1}}{z}, Q^2\right) D_g^{h_2}\left(\frac{z_{h_2}}{1-z}, Q^2\right)
\end{aligned}
$$

$$+ (1 \leftrightarrow 2) + \int_{z_{h_1}+z_{h_2}}^{1} \frac{\mathrm{d}z}{z^2} \gamma_{qg}(z) D_g^{h_1 h_2}\left(\frac{z_{h_1}}{z}, \frac{z_{h_2}}{z}, Q^2\right)\Bigg] , \quad (4)$$

that is similar to the DGLAP evolution equations for single hadron jet fragmentation functions. However, there are extra contributions in the above equation that are proportional to convolution of two single hadron fragmentation functions. These correspond to independent fragmentation of both daughter partons after the parton split in the radiative correction. Here γ_{qq} is the normal parton splitting function and $\tilde{\gamma}_{qq}$ is the same function without the virtual corrections. For numerical solution of the DGLAP evolution equations of the dihadron fragmentation, results from JETSET [30] at a given scale are used. The evolved dihadron fragmentation functions from the DGLAP equations at higher scales agree with the Monte Carlo results very well [29]. It will be useful to compare actual experimental data when they become available.

Since the induced bremsstrahlung in medium is similar to that in vacuum, medium modification to the jet fragmentation functions should resemble the radiative corrections in vacuum that lead to the DGLAP evolution equations. Therefore, it is not surprising that the medium modification to the dihadron fragmentation functions has a form identical to the above DGLAP evolution equations [31]. These medium modifications depend on the same gluon correlation functions as in the modification to the single hadron fragmentation functions. Therefore, in the numerical calculation of the medium modification of dihadron fragmentation functions, there are no additional parameters involved. The predicted results are in good agreement with HERMES data [31]. We find that most of the nuclear modification is manifest in the single hadron fragmentation functions. Since dihadron fragmentation functions already contain the information of the single hadron fragmentation function, the modification to the remaining correlated distribution is very small. So the normalized correlation $D_q^{h_1h_2}(z_1, z_2)/D_q^{h_1}(z_1)D_q^{h_2}(z_2)$ has a much smaller nuclear modification as compared to the single hadron fragmentation functions. This also explains why the same-side two-hadron correlation in central heavy-ion collisions remains approximately the same as in $p+p$ collisions while the back-side is completely suppressed [21]. However, because of trigger bias, one might sample different values of z_1 and z_2 in $\mathrm{Au}+\mathrm{Au}$ and $p+p$ collisions. This could lead to an apparent change of the dihadron correlation [31].

5 Jet fragmentation and quark recombination

During the propagation and interaction inside a deconfined hot partonic medium, a fast parton has not only induced gluon radiation but also induced absorption of the surrounding thermal gluons. This leads to a stronger energy dependence of the net energy loss for an intermediate energy parton [32]. In principle, one can consider such processes of detailed balance as parton recombination and they can continue until the hadronization of the bulk partonic matter. Eventually, during the hadronization, partons from the jet can combine with those from the medium to form final hadrons. Indeed, there exists already some evidence for the parton recombination in the experimental data on the final hadron spectra in heavy-ion collisions at RHIC. At intermediate $p_\mathrm{T} = 2$–$4\,\mathrm{GeV}/c$, the suppression of baryons due to jet quenching is significantly smaller than mesons leading to a baryon to meson ratio larger than 1, about a factor of 5 increase over the value in $p+p$ collisions [33]. On the other hand, the azimuthal anisotropy of the baryon spectra is higher than that of meson spectra. Such a flavor dependence of the nuclear modification of the hadron spectra and their azimuthal anisotropy is not consistent with a picture of pure parton energy loss. The most striking revelation of the underlying mechanism comes from the empirical observation of the scaling behavior between the azimuthal anisotropy of baryons and mesons [34], $v_2^M(p_\mathrm{T}/2)/2 = v_2^B(p_\mathrm{T}/3)/3$, inspired by a schematic model of constituent quark recombination of hadron production.

Many quark recombination models [35,36] are successful in describing the observed flavor dependence of the nuclear modification of hadron spectra at intermediate p_T. These models generally have three different contributions to the final hadron spectra. They include recombination of the thermal quarks in the bulk matter into hadrons which dominate low p_T spectra and recombination between thermal quarks and quarks from high transverse momentum jets that are responsible for intermediate p_T hadrons. They all assume a thermal distribution for the medium quarks and employ the constituent quark model for the hadron wavefunctions which determine the recombination probabilities. However, current models differ in the determination of the constituent quark distributions from high-p_T jets and there exist ambiguities in the connection between partons from pQCD hard processes and constituent quarks that form the final hadrons. Perhaps the most consistent treatment of the problem is the model by Hwa and Yang [35]. In this model, quark recombination processes are traced back to parton fragmentation processes in vacuum. They consider the initial produced hard partons that will evolve into a shower of constituent quarks which then recombine to form the final hadrons in the parton fragmentation process. The recombination of the shower quarks of the parton jets with the medium quarks in heavy-ion collisions can be carried out straightforwardly given both the shower and medium quark distributions. Since this is a phenomenological model, the nuclear modification of the jet shower quark distributions and their QCD evolution cannot be calculated in the Hwa–Yang model. The model has to rely on fitting to the experimentally measured hadron spectra to obtain the corresponding nuclear modified shower quark distributions for each centrality of heavy-ion collisions and correlations between shower quarks are completely neglected.

We have made a first attempt to derive the quark recombination model for jet fragmentation functions from the field theoretical formulation and the constituent quark model of hadrons [37]. Within the constituent quark model, we consider the parton fragmentation as a two stage process. The initial parton first evolves into a shower of constituent quarks that subsequently will combine with each other to form the final hadrons. Since constituent quarks are non-perturbative objects in QCD similarly as hadrons, the conversion of hard partons into showers of constituent quarks is not calculable in pQCD. However, we can define constituent quark distributions in a jet as overlapping matrices of the parton field operator and the constituent quark states, similarly as the definition of the hadron fragmentation functions.

Given a hadron's (a meson for example) wavefunction in the constituent quark model,

$$|p_h\rangle = \int \frac{\mathrm{d}^2 k_{1\perp}}{2(2\pi)^3} \frac{\mathrm{d}x_1}{\sqrt{x_1(1-x_1)}} \times \varphi_h(k_{1\perp}, x_1; -k_{1\perp}, 1-x_1)|k_{1\perp}, x_1; -k_{1\perp}, 1-x_1\rangle, \quad (5)$$

and neglecting interferences between recombination of a quark and anti-quark pair with different momentum, one can rewrite the single inclusive meson fragmentation function as a convolution of the diquark distribution functions and the recombination probability,

$$\begin{aligned} D_q^h(z_h) &\equiv \frac{z_h^3}{2} \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \delta\left(z_h - \frac{p_h^+}{p^+}\right) \int \mathrm{d}^4 x \mathrm{e}^{-\mathrm{i}p\cdot x} \\ &\times \mathrm{Tr}\left[\frac{\gamma^+}{2p_h^+} \sum_S \langle 0 |\psi(0)| S, p_h\rangle \langle p_h, S |\overline{\psi}(x)| 0\rangle\right] \\ &\approx C_h \int_0^{z_h} \frac{\mathrm{d}z_1}{2} R_h\left(0_\perp, \frac{z_1}{z_M}\right) F_q^{q_1\bar{q}_2}(z_1, z_h - z_1), \quad (6) \end{aligned}$$

where C_h is a constant representing contributions from interference processes. The recombination probability is determined by the hadrons' wavefunction,

$$R_h\left(k_{1\perp}, \frac{z_1}{z_h}\right) \equiv \left|\varphi_h\left(k_{1\perp}, \frac{z_1}{z_h}; -k_{1\perp}, 1 - \frac{z_1}{z_h}\right)\right|^2, \quad (7)$$

and the double constituent quark distribution function is defined as the overlapping matrix between the current quark operators and the intermediate constituent quark states,

$$\begin{aligned} &F_q^{q_1\bar{q}_2}(z_1, z_2) \\ &= \frac{z_h^4}{2z_1 z_2} \int^{\Lambda} \frac{\mathrm{d}^2 k_{1\perp}}{2(2\pi)^3} \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \int \mathrm{d}^4 x \delta\left(z_h - \frac{p_h^+}{p^+}\right) \\ &\times \mathrm{e}^{-\mathrm{i}p\cdot x} \mathrm{Tr}\left[\frac{\gamma^+}{2p_h^+} \sum_S \langle 0 |\psi(0)| S, k_1, k_2\rangle \right. \\ &\left. \times \langle k_2, k_1, S |\overline{\psi}(x)| 0\rangle\right]. \quad (8) \end{aligned}$$

Here, $p_h = k_1 + k_2$ and $z_h = z_1 + z_2$. Λ is the cutoff for the intrinsic transverse momentum of the constituent quarks inside a hadron, as provided by the hadron wavefunction. The above definition of the diquark distribution function in a fragmenting parton jet has exactly the same form as the dihadron fragmentation functions [29]. One can similarly express the parton fragmentation functions for baryons in terms of triquark distribution functions. This is similar in spirit to the Hwa–Yang recombination model. Given the form of the hadrons' wavefunction in the constituent quark model, one can in principle extract constituent quark distribution functions from the measured jet fragmentation functions. Furthermore, within this framework, one can also derive the DGLAP evolution equations for the diquark distribution functions,

$$\begin{aligned} &Q^2 \frac{\mathrm{d}}{\mathrm{d}Q^2} F_q^{q_1\bar{q}_2}(z_1, z_2, Q^2) \\ &= \frac{\alpha_{\mathrm{s}}(Q^2)}{2\pi} \int_{z_1+z_2}^1 \frac{\mathrm{d}z}{z^2} \left[\gamma_{qq}(z) F_q^{q_1\bar{q}_2}\left(\frac{z_1}{z}, \frac{z_2}{z}, Q^2\right)\right. \\ &\left. + \gamma_{qg}(z) F_g^{q_1\bar{q}_2}\left(\frac{z_1}{z}, \frac{z_2}{z}, Q^2\right)\right], \quad (9) \\ &Q^2 \frac{\mathrm{d}}{\mathrm{d}Q^2} F_g^{q_1\bar{q}_2}(z_1, z_2, Q^2) \\ &= \frac{\alpha_{\mathrm{s}}(Q^2)}{2\pi} \int_{z_1+z_2}^1 \frac{\mathrm{d}z}{z^2} \left[\gamma_{gq}(z) F_s^{q_1\bar{q}_2}\left(\frac{z_1}{z}, \frac{z_2}{z}, Q^2\right)\right. \\ &\left. + \gamma_{gg}(z) F_g^{q_1\bar{q}_2}\left(\frac{z_1}{z}, \frac{z_2}{z}, Q^2\right)\right]. \quad (10) \end{aligned}$$

Note that the above equations are a little different from the DGLAP evolution equations for dihadron fragmentation functions. There is no contribution from independent fragmentation in the diquark distribution. This is because the diquark distribution functions defined in the context of quark recombination are only for two quarks whose relative transverse momentum is limited by the wavefunction. Therefore, change of the initial momentum scale does not lead to variation of phase space available for the diquark from the independent fragmentation in the final states. One can show that combining the above evolution equations for the double constituent quark distribution functions with the expression of jet fragmentation function in (14), the DGLAP evolution equations for single inclusive hadron fragmentation functions can be recovered.

The above reformulation of the jet fragmentation functions does nothing to simplify the complexity of jet hadronization. However, extending the formalism to finite temperature, we can automatically derive the contributions from recombination between shower and thermal quarks in addition to soft hadron production from recombination of thermal quarks and leading hadron from recombination of shower quarks. The shower and thermal quark recombination involves single quark distribution functions which are related to the diquark distributions through sum rules. Therefore, one can consistently describe three different processes within this formalism. Since the single and diquark distribution functions are defined at finite temperature which are different from the corresponding vacuum distributions, one can also consistently take into account the parton energy loss and detailed balance effect for jet fragmentation inside a thermal medium.

6 Modified jet fragmentation due to quark recombination in medium

One can study the fragmentation of a parton jet in medium simply by replacing the vacuum expectation in the S matrix of the processes or the operator definition of the parton fragmentation functions by the thermal expectation values, $\langle 0|\mathcal{O}|0\rangle \to \langle\langle \mathcal{O}\rangle\rangle$,

$$\langle\langle \mathcal{O}\rangle\rangle = \frac{\mathrm{Tr}[\mathrm{e}^{-\hat{H}/T}\mathcal{O}]}{\mathrm{Tr}\ \mathrm{e}^{-\hat{H}/T}}\,, \tag{11}$$

where $\hat{H}$ is the Hamiltonian operator of the system and T is the temperature. Therefore, the single hadron fragmentation at finite temperature for a quark is defined as

$$\begin{aligned}&\tilde{D}_q^h(z_h,p^+)\\ &= \frac{z_h^3}{2}\int\frac{\mathrm{d}^4p}{(2\pi)^4}\delta\left(z_h-\frac{p_h^+}{p^+}\right)\int \mathrm{d}^4x\mathrm{e}^{-\mathrm{i}p\cdot x}\\ &\quad\times\mathrm{Tr}\left[\frac{\gamma^+}{2p_h^+}\sum_S\langle\langle\psi(0)|S,p_h\rangle\langle p_h,S|\overline{\psi}(x)\rangle\rangle\right]\,,\end{aligned} \tag{12}$$

where p_h and p are the four-momentum of the hadron and the initial parton, respectively. After making all possible contractions between the final constituent quark states with the thermal intermediate states, one can obtain three distinctive contributions to the above fragmentation function in medium,

$$\begin{aligned}&\tilde{D}_q^h(z_h,p^+)\\ &= \tilde{D}_q^{h(\mathrm{SS})}(z_h,p^+)+\tilde{D}_q^{h(\mathrm{ST})}(z_h,p^+)+\tilde{D}_q^{h(\mathrm{TT})}(z_h,p^+)\,.\end{aligned} \tag{13}$$

The first contribution,

$$\begin{aligned}&\tilde{D}_q^{h(\mathrm{SS})}(z_h)\\ &\approx C_h\int_0^{z_h}\frac{\mathrm{d}z_1}{2}R_h\left(0_\perp,\frac{z_1}{z_h}\right)\tilde{F}_q^{q_1\bar{q}_2}(z_1,z_h-z_1,p^+)\,,\end{aligned} \tag{14}$$

normally referred [35] to as "shower–shower" quark recombination comes from recombination of constituent quarks from within the parton jet. It has exactly the same form as the parton fragmentation functions in vacuum [see (14)] in the framework of quark recombination, except that the diquark distribution function $\tilde{F}_q^{q_1\bar{q}_2}(z_1,z_2)$ are now also modified by the medium. Its definition is similar to that in vacuum in (8), but the vacuum expectation is replaced by thermal average. These modified diquark distribution functions should in principle contain effects of multiple scattering, induced gluon radiation and absorption, in the same way as the modification of hadron fragmentation functions in a thermal medium [38].

The second term in the modified fragmentation function,

$$\begin{aligned}&\tilde{D}_q^{h(\mathrm{ST})}(z_h,p^+)\\ &= \int_0^{z_h}\frac{\mathrm{d}z_q}{z_h}\int\frac{\mathrm{d}^2q_\perp}{2(2\pi)^3}\frac{R_h(q_\perp,z_q/z_h)}{(1-z_q/z_h)^2}\\ &\quad\times\Big[f_q(q_\perp,z_qp^+)\tilde{F}_q^{\bar{q}}(z_h-z_q)\\ &\qquad+f_{\bar{q}}(q_\perp,z_qp^+)\tilde{F}_q^{q}(z_h-z_q)\Big]\,,\end{aligned} \tag{15}$$

is from recombination between a constituent quark (anti-quark) from the parton jet and an anti-quark (quark) from the medium. This is often referred to as "thermal–shower" quark recombination. Here, f_q and $f_{\bar{q}}$ are thermal quark distributions, and $\tilde{F}_q^q(z)$ and $\tilde{F}_q^{\bar{q}}(z)$ are single constituent quark or anti-quark distributions of the fragmenting parton jet in a thermal medium defined similarly as in the vacuum, except that the vacuum expectation values are replaced again by thermal averaged expectation. They should be different from the corresponding quark distributions in vacuum because of multiple scattering, induced gluon bremsstrahlung and parton absorption.

The final term in (13),

$$\begin{aligned}&\tilde{D}_q^{h(\mathrm{TT})}(z_h,p^+)\\ &= Vp^+\int\frac{\mathrm{d}^2p_{h\perp}}{(2\pi)^3}\int_0^{z_h}\frac{\mathrm{d}z_q}{2z_h}\int\frac{\mathrm{d}^2q_\perp}{(2\pi)^3}\\ &\quad\times f_q(q_\perp,z_qp^+)f_{\bar{q}}(p_{h\perp}-q_\perp,(z_h-z_q)p^+)\\ &\quad\times R_h\left(q_\perp,\frac{z_q}{z_h}\right)\,,\end{aligned} \tag{16}$$

comes from recombination of two thermal constituent quarks. Here V is the total volume of the whole thermal system. Since hadron production from the thermal quark recombination is not correlated with the parton jet and its fragmentation, the above expression is a little bit unnatural. One should be able to rewrite it in terms of the invariant hadron spectrum from thermal quark recombination,

$$\begin{aligned}&(2\pi)^3\frac{\mathrm{d}N^{h(\mathrm{TT})}}{\mathrm{d}p_h^+\mathrm{d}^2p_{h\perp}}\\ &= V\int_0^1\mathrm{d}x_q\int\frac{\mathrm{d}^2q_\perp}{2(2\pi)^3}f_q(q_\perp,x_qp_h^+)\\ &\quad\times f_{\bar{q}}(p_{h\perp}-q_\perp,(1-x_q)p_h^+)R_h(q_\perp,x_q)\,,\end{aligned} \tag{17}$$

which is not correlated and therefore do not depend on the parton jet fragmentation. The above expression also coincides with the results from other recombination models [36].

7 Conclusions

In this talk, I reviewed some new developments in the study of modified single and dihadron fragmentation functions in a dense medium and their applications to heavy-ion collisions. All of them will help to provide further tests of the picture of parton energy loss and jet quenching and enable a more detailed characterization of the dense medium those jets probe. The mass dependence of the gluon formation time from the heavy quark leads to a unique change of the medium size dependence of the heavy quark energy loss, from linear to quadratic, when the initial quark energy and the momentum scale are varied. The so-called "dead-cone" effect, also caused by the heavy quark mass, in addition reduces the total heavy quark energy loss. The form of the heavy quark fragmentation function into heavy quark mesons in vacuum, which is peaked at large fractional momentum z, leads to a medium modification that is different for light hadrons from massless partons. Since one can identify heavy quark mesons, one can use them to tag heavy quark propagation and study the difference between quark and gluon energy loss [39]. One can also use the energy dependence of the suppression of single inclusive hadron spectra to test the difference in quark and gluon energy loss due to the non-Abelian gauge interaction. For fixed $p_{\rm T}$ the fraction of initial jets changes from quark to gluon-dominated partons and the different energy losses of quarks and gluons will result in a unique energy dependence.

Finally, I also discussed the formulation of fragmentation functions in a quark recombination picture within a constituent quark model. Given the hadron's wavefunction in the constituent quark model and neglecting interference effects, we have shown that hadron fragmentation functions can be expressed as the convolution of the recombination probability (given by the hadron's wavefunction) and the constituent (or shower) quark distribution of the jet. The constituent quark distributions are defined as the overlapping matrices between parton fields and the final constituent quark states, just like hadron fragmentation functions as overlapping matrices between parton fields and final hadrons. We have derived the DGLAP evolution equations for the quark distribution functions. We then extended the formalism to include the medium effect within the framework of field theory at finite temperature. One naturally arrived at three distinctive contributions from recombination between shower constituent quarks, shower–thermal and thermal–thermal quarks, as has been proposed by previous recombination models.

Since parton energy loss is only sensitive to the initial color charge (or gluon) density of the medium, the observed jet quenching points us to an enormously high initial gluon density created in the central Au + Au collisions at RHIC. However, parton energy loss does not distinguish confined and deconfined matter. Quark recombination between jet shower quarks and thermal quarks on the other hand is a process of quark interaction that crosses the hadronic boundary. If proven, the combined signal of quark recombination and thermalization will lead to the unambiguous conclusion of deconfinement of the produced dense matter.

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